The Influence of Discontinuities on Elastic and Mechanical Properties of Composite Materials Reinforced with Woven Carbon, Carbon-kevlar and Kevlar

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In this paper we have studied the influence of discontinuities on elastic and mechanical properties of composite materials reinforced with woven Carbon, Carbon-Kevlar and Kevlar. In addition, we have studied the way variations of the volume proportion of reinforcement influences elasticity modulus and tensile strength for the studied composite materials. In order to appreciate the property difference between different areas of the composite material and also the dimensions of the defective areas, a relative uniformity influence coefficient was introduced offering us a tool for assessing the mechanical behaviour of the studied composite compared with a reference composite. We have also performed experimental validations of the obtained theoretical results.

Keywords: composite materials, relative uniformity coefficient, elasticity modulus, tensile strength

Traditional laminated composites reinforced with fibres are formed of multiple lamina, each being a pattern of materials and directions of the reinforcement fibres. The possibility of designing traditional lamina is limited to the selection of materials for components and the orientation of the fibres of individual layers, as well as the sequence of layers in the laminate.

“Mosaic” type composites differ from traditional laminates. Each layer is made up of several pieces with particular orientation, length and distribution of fibres. A simple joining of such layers may lead to a composite with a modulus of elasticity close to a continuous unidirectional reinforced composite, but has a tensile strength reduced by 50% [1-2]. Using such elements in regular assemblies that intertwine has increased tensile strength up to 90% from that of composites with continuous reinforcement [3-4]. In [5-6] it has been showed that reducing cohesion between adjacent elements may reveal a mechanism for slowing down cracking, resulting in composite materials with improved fault tolerance.

The elastic and strength properties of composite materials can be influenced by various defects that can arise due to the manufacturing process. This is important for the production of large series of parts made out of fibres reinforced composite materials, because the fibres in the matrix should be distributed unevenly. In the composites reinforced with filaments or fibres, a grouping phenomenon of the fibres often occurs. In [7] the effect of these grouping phenomena is studied and the influence these have on the real thermal conductivity of composites. An analytical model was developed based on the theory of mediation, an effective model for calculating the real thermal conductivity of composites that takes into account the unevenness occurring in the distribution of the reinforcement fibres. In [8] the study regarding the effect of uneven distribution of fibres on the thermal expansion of the graphite/copper composite determined that the irregularity affects the thermal expansion particularly in the longitudinal direction.

In practice, the random microscopic variations of properties that make up the composite material are not uniform. Stochastic homogenization methods for predicting mechanical characteristics, which treats the material as a homogeneous and heterogeneous composite, have been developed in recent years. There have been developed methods based on the Monte Carlo simulation [9-10], based on the disturbance analysis and homogenization with finite element method [11] or an equivalent material detection [12]. In [13] a problem of homogenization is discussed and it is used to evaluate the elastic characteristics for a particular composite material reinforced with particles, which takes into account the uneven distribution of the geometric properties of component materials and also their random variation.

In [19] unevenness occurring in the composite rods reinforced with glass fibre fabric were studied. A coefficient that considers unevenness in composite bars that have two areas with different proportions of reinforcement volume was presented.

Theoretical aspects

Let us consider a constant rectangular cross section composite rod of length **l**, formed out of 2n symmetrical layers in relation with the median plane (fig. 1). The “x” axis represents the longitudinal axis in the median plane, and the ‘’z’’ axis is normal to the median plane.

We shall consider that all layers have the same thickness $g / 2n$, where g is the total thickness of the rod.

If the rod is subjected to a tensile load, the symmetry we won’t allow buckling or torsion phenomena occur. Therefore we will acknowledge that $\sigma_{xy} = \sigma_{yx} = \sigma_{yy} = \sigma_{zz} = 0$. The different than zero loads must verify the Cauchy equation:

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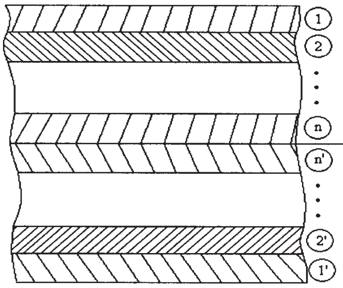

Fig. 1

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} = 0. \qquad (1)$$

If each layer has the same elastic properties over the entire length of the rod, than the load σ_{xx} is constant at any point of the layer and the tangential load σ_{xz} is zero. If, however, there are variations of elastic properties in a layer, the assumed non-zero tensions σ_{xx} and σ_{xz} will be dependent of variables *x* and *z*.

We will consider that in one layer *i* around the section $x = x_p$, there is a length u_p where there is a variation of elastic properties (fig. 2).

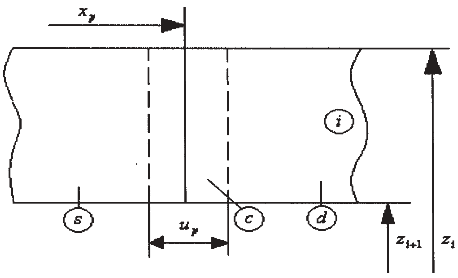

Fig. 2.

In the areas *s* and *d* we will consider that the elastic properties are constant, variations existing only in the area *c*. In the hypothesis that the specific strain ε_{xx} depends only of variable x than in the area *s* we have a specific strain of the following type:

$$\varepsilon_{xx}^{(s)} = \sigma \cdot \frac{n}{\sum_{k=1}^{n} E^{(k;s)}}, \qquad (2)$$

where σ is the medium load in the cross section

$$\sigma = \frac{F}{b \cdot g}, \qquad (3)$$

where:

F - tensile force;

b - rod width;

$E^{(k;s)}$ is the modulus of elasticity of the material in layer k, area s.

In area *s*, layer *j*, the normal load has the following expression:

$$\sigma_{xx}^{(j;s)} = \sigma \cdot \frac{n E^{(j;s)}}{\sum_{k=1}^{n} E^{(k;s)}}. \qquad (4)$$

In area *d* we will have the following specific strain:

$$\varepsilon_{xx}^{(d)} = \sigma \cdot \frac{n}{\sum_{k=1}^{n} E^{(k;d)}}, \qquad (5)$$

where $E^{(k;d)}$ is the modulus of elasticity of the material in layer *k*, area *d*

In area *d* , layer *j*, the normal load has the following expression:

$$\sigma_{xx}^{(j;d)} = \sigma \cdot \frac{n E^{(j;d)}}{\sum_{k=1}^{n} E^{(k;d)}}. \qquad (6)$$

In area *c* we will consider that the specific strain $\varepsilon_{xx}^{(c)}$ has a linear variation. By having the conditions:

$$\varepsilon_{xx}^{(c)}\left(x_p - \frac{u_p}{2}\right) = \varepsilon_{xx}^{(s)}, \quad \varepsilon_{xx}^{(c)}\left(x_p + \frac{u_p}{2}\right) = \varepsilon_{xx}^{(d)}, \qquad (7)$$

we will obtain

$$\varepsilon_{xx}^{(c)}(x) = \frac{\varepsilon_{xx}^{(d)} - \varepsilon_{xx}^{(s)}}{u_p}\left(x - x_p\right) + \frac{\varepsilon_{xx}^{(d)} + \varepsilon_{xx}^{(s)}}{2}. \qquad (8)$$

In area *c* we will consider linear variation for the longitudinal elasticity modulus, $E^{(j,c)}$ for each layer $j=\overline{1,n}$. By having the conditions:

$$E^{(j;c)}\left(x_p - \frac{u_p}{2}\right) = E^{(j;s)}, \quad E^{(j;c)}\left(x_p + \frac{u_p}{2}\right) = E^{(j;d)}, \qquad (9)$$

we will obtain

$$E^{(j;c)}(x) = \frac{E^{(j;d)} - E^{(j;s)}}{u_p}\left(x - x_p\right) + \frac{E^{(j;d)} + E^{(j;s)}}{2}. \qquad (10)$$

We can observe that for the layers where we don't have a variation of the elasticity modulus we will have:

$$E^{(j;c)} = E^{(j;s)} = E^{(j;d)} = E^{(j)}, \qquad (11)$$

$E^{(j)}$ in the elasticity modulus of layer *j*.

The normal load in layer *j*, area *c*, is:

$$\sigma_{xx}^{(j;c)} = E^{(j;c)}(x) \cdot \varepsilon_{xx}^{(j;c)} = \frac{\left(\varepsilon_{xx}^{(d)} - \varepsilon_{xx}^{(s)}\right)\left(E^{(j;d)} - E^{(j;s)}\right)}{u_p^2}\left(x - x_p\right)^2 + \frac{1}{u_p}\left[\varepsilon_{xx}^{(d)}E^{(j;d)} - \varepsilon_{xx}^{(s)}E^{(j;s)}\right]\left(x - x_p\right) + \frac{\left(\varepsilon_{xx}^{(d)} + \varepsilon_{xx}^{(s)}\right)\left(E^{(j;d)} - E^{(j;s)}\right)}{4}. \qquad (12)$$

For the layers where we don't have variation of the elasticity modulus we will have:

$$\sigma_{xx}^{(j;c)} = \frac{1}{u_p}E^{(j)}\left(\varepsilon_{xx}^{(d)} - \varepsilon_{xx}^{(s)}\right)\left(x - x_p\right) + \frac{1}{2}E^{(j)}\left(\varepsilon_{xx}^{(d)} + \varepsilon_{xx}^{(s)}\right) \qquad (13)$$

From condition (1) we can determine the tangential load. σ_{xz}. For layer *j*, area *c* we obtain:

$$\sigma_{xz}^{(j;c)}(x;z) = \tau^{(j)}(x) - f_j(x) \cdot z, \qquad (14)$$

where:

$$f_j(x) = \frac{\left(\varepsilon_{xx}^{(d)} - \varepsilon_{xx}^{(s)}\right)\left(E^{(j;d)} - E^{(j;s)}\right)}{u_p^2} \cdot 2\left(x - x_p\right) + \frac{1}{u_p}\left(\varepsilon_{xx}^{(d)}E^{(j;d)} - \varepsilon_{xx}^{(s)}E^{(j;s)}\right) \qquad (15)$$

For the layers where we don't have variation of the modulus of elasticity:

$$\sigma_{xz}^{(j;c)}(x;z) = \tau^{(j)}(x) - \frac{1}{u_p}E^{(j)}\left(\varepsilon_{xx}^{(d)} - \varepsilon_{xx}^{(s)}\right)z. \qquad (16)$$

The terms $\tau^{(j)}(x)$, $j = \overline{1,n}$, are obtained from the tangential load continuity conditions on the separation surfaces between layers. Because exterior surfaces do not subject to the condition $\sigma_{xz}^{(j;c)}\left(x; \frac{g}{2}\right) = 0$, we will have:

$$\tau^{(1)}(x)=f_1(x)\cdot\frac{g}{2}=\left[\frac{\left(\varepsilon_{xx}^{(d)}-\varepsilon_{xx}^{(s)}\right)\left(E^{(1;d)}-E^{(1;s)}\right)}{u_p^2}\cdot\right.$$

$$\left.2\left(x-x_p\right)+\frac{1}{u_p}\left(\varepsilon_{xx}^{(d)}E^{(1;d)}-\varepsilon_{xx}^{(s)}E^{(1;s)}\right)\right]\frac{g}{2}. \qquad (17)$$

Out of the continuity conditions

$$\sigma_{xz}^{(j+1;c)}\left(x;\frac{g}{2}\left(1-\frac{j}{n}\right)\right)=\sigma_{xz}^{(j;c)}\left(x;\frac{g}{2}\left(1-\frac{j}{n}\right)\right) \qquad (18)$$

Resulting

$$\tau^{(k)}(x)=\tau^{(1)}(x)+\frac{g}{2}\left(f_k(x)-f_1(x)\right)+\frac{g}{2n}\sum_{j=1}^{k-1}j\left(f_{j+1}(x)-f_j(x)\right). \qquad (19)$$

The rod elongation is calculated with the expression:

$$\Delta l=\int_0^l\varepsilon_{xx}dx=\sum_{(p)}\left[\varepsilon_{xx}^{(s)}l_s+\frac{\varepsilon_{xx}^{(s)}+\varepsilon_{xx}^{(d)}}{2}u_p+\varepsilon_{xx}^{(d)}l_d\right]_{(p)}, \qquad (20)$$

the summation being made after the number of areas with discontinuities where l_s is the length of area *s* and l_d is the length of area *d*. If $u_p \rightarrow 0$, than:

$$\Delta l=\sum_{(p)}\left[\varepsilon_{xx}^{(s)}l_s+\varepsilon_{xx}^{(d)}l_d\right]_{(p)}. \qquad (21)$$

We can calculate an average modulus of elasticity for the rod's material, by assuming that we have a homogenous rod and we will get:

$$E=\frac{l\cdot\sigma}{\Delta l}=\frac{l\cdot\sigma}{\sum_{(p)}\left[\varepsilon_{xx}^{(s)}l_s+\varepsilon_{xx}^{(d)}l_d\right]_{(p)}}. \qquad (22)$$

By replacing $\varepsilon_{xx}^{(s)}$ and $\varepsilon_{xx}^{(d)}$ given by (2) and (5) we will have the following formula for calculating the modulus of elasticity:

$$E=\frac{l}{n\sum_{(p)}\left[\frac{l_s}{\sum_{k=1}^{n}E^{(k;s)}}+\frac{l_d}{\sum_{k=1}^{n}E^{(k;d)}}\right]_{(p)}}. \qquad (23)$$

Damage of the specimen takes place when a layer reaches the breaking strength of the material.

If the loading applied to the specimen increases, two phenomena can appear:

- immediately after the layer deteriorates, the loads from other layers reach the ultimate tensile strength and the specimen rift occurs;
- the other layers take over the stresses and the tensile force increases until it reaches the breaking point for the next layer.

In order to understand this fracture mechanism, we consider that in layer *i*, area *s*, the material has a modulus of elasticity $E^{(i,s)} = E_1$, the tensile strength $\sigma_r^{(i,s)} = \sigma_{r1}$ and in the rest of that particular layer and other layers, the material has a modulus of elasticity of $E^{(k;s)} = E^{(k;d)} = E_2$ and a tensile strength $\sigma_r^{(k;s)} = \sigma_r^{(k;d)} = \sigma_{r2}$.

In these conditions, the loads in area *s*

$$\sigma^{(i;s)}=\frac{\sigma\cdot n\cdot E_1}{E_1+(n-1)E_2}, \qquad (24)$$

$$\sigma^{(j;s)}=\frac{\sigma\cdot n\cdot E_2}{E_1+(n-1)E_2};\ j\neq i. \qquad (25)$$

In area *d* the stress states are:

$$\sigma^{(j;d)}=\sigma\,. \qquad (26)$$

We shall suppose that in layer *i*, area *s* terioration of the specimen takes place in this area. If layer *i*, area *s* breaks, than the other layers will take over the load. Thus, the load in area *s* will be:

$$\sigma^{(j;s)}=\frac{\sigma\cdot n}{n-1},\ j\neq i, \qquad (27)$$

while in area *d* the load is given by (26).

Because $\sigma^{(j;s)}$. $\sigma^{(j;d)}$, we will obtain that the breaking of the specimen will occur in area *s* and the tensile strength of the specimen is:

$$\sigma_r=\frac{n-1}{n}\sigma_{r2}. \qquad (28)$$

By extension, if in area *s* we have *t* layers with discontinuities, the tensile strength is:

$$\sigma_r=\frac{n-t}{n}\sigma_{r2}. \qquad (29)$$

In [16-17] a coefficient of uniformity was defined for a composite by taking into consideration the tensile strength and the modulus of elasticity of the fibres used as reinforcement.

In a similar manner we will introduce a relative uniformity coefficient that takes into account the properties of a reference material:

$$c_{ur}=\frac{\sigma_r\cdot E_{ref}}{\sigma_{ref}\cdot E}, \qquad (30)$$

where:

- σ_ρ is the tensile strength of the material under study;
- E is the modulus of elasticity of the material under study;
- σ_{ref} is the tensile strength of the reference material;
- E_{ref} is the modulus of elasticity of the reference material.

In the case of the composites, the reference material is made out of layers without discontinuities.

Experimental part

We made plates of composite materials with epoxy resin matrix (Resoltech 1050 with Resoltech 1058 hardener), reinforced with carbon fibre fabric, Kevlar fibre fabric and mixed fabric of carbon fibre and Kevlar. Each plate is formed out of 10 layers of fabric, disposed symmetrically relative to the median plane. Discontinuities of different sizes are present in the above mentioned two or four layer plates. For each of the three types of fabric we have made reference plates, all with ten layers arranged symmetrically, but without any discontinuities.

From each plate, sets of samples were taken. The sets of reinforced carbon fabric specimens were abbreviated as follows C00, C20, C22, C24, C40, C42, C44, the reinforced Kevlar fabric were abbreviated K00, K20, K24, K40, K42, K44, and the sets of specimens reinforced with carbon fabric and Kevlar were abbreviated CK00, CK20, CK22, CK24, CK40, CK42, CK44. The first figure is the number of layers with discontinuities, and the second figure is the length of the discontinuity in centimetres. For example K42 is the set of specimens reinforced with Kevlar fabric, with four layers of discontinuities and length of two centimetres of discontinuity for each layer.

The specimens were subjected to tensile loads. In this case, the calibrated length of specimen subjected to tests was 10 cm.

The strain-stress curve for the specimens in sets C00, C22, C44 were presented in figure 3.

The strain-stress curve for the specimens in sets K00, K22, K44 were presented in figure 4.

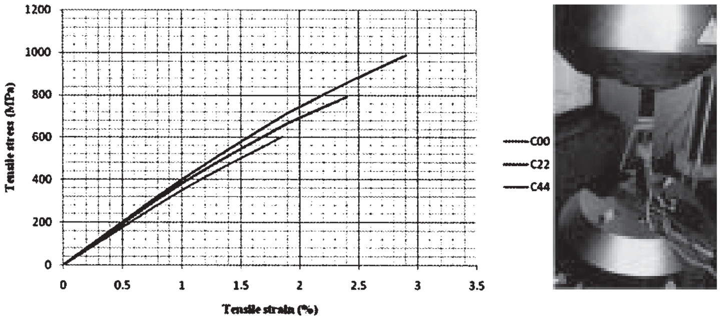

Fig. 3

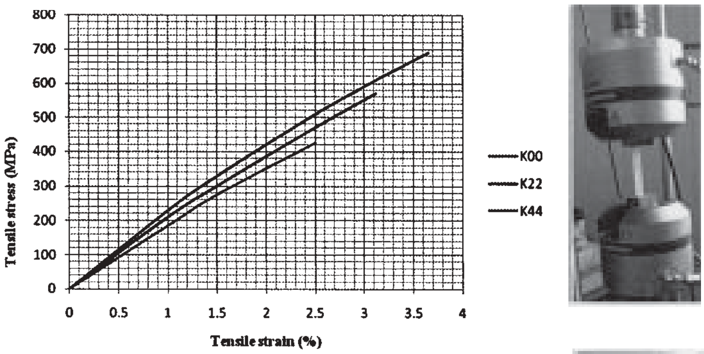

Fig. 4

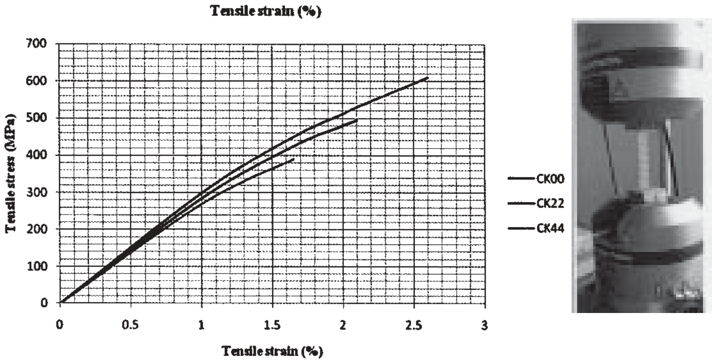

Fig. 5

Table 1

Set of specimens	Modulus of elasticity (MPa)		Tensile strength (MPa)		Relative uniformity coefficient
	Theoretical	Experimental	Theoretical	Experimental	
C20	44079	43155	792	795	0.820
C22	41980	41691	792	770	0.822
C24	40072	39587	792	775	0.871
C40	44079	43312	594	608	0.625
C42	38893	38290	594	604	0.720
C44	34799	35549	594	597	0.748

Table 2

Set of specimens	Modulus of elasticity (MPa)		Tensile strength (MPa)		Relative uniformity coefficient
	Theoretical	Experimental	Theoretical	Experimental	
K20	26001	26067	552	556	0.804
K22	24763	25111	552	553	0.830
K24	23637	23338	552	566	0.913
K40	26001	26122	414	405	0.584
K42	22942	23876	414	413	0.652
K44	20527	22126	414	426	0.726

The strain-stress curve for the specimens in sets CK00, CK22, CK44 were presented comparatively in figure 5.

We have chosen these particular sets of specimens because they highlight the variations of the modulus of elasticity and the ultimate tensile strength.

For the sets of reference specimens we have obtained the following representative results:

- for set C00
  - -modulus of elasticity; E_{ref} = 44079 MPa
  - -tensile strength; σ_{ref} = 990 MPa
- for set K00
  - -modulus of elasticity; E_{ref} = 26001MPa
  - -tensile strength; σ_{ref} = 690 MPa
- for set CK00
  - -modulus of elasticity; E_{ref} = 33583 MPa
  - -tensile strength. σ_{ref} = 625MPa

In table 1, theoretical and experimental results of the modulus of elasticity, ultimate tensile strength and relative uniformity coefficient for reinforced specimens with carbon fabric are presented.

For calculating the theoretical values of the material with discontinuities we used the experimental results obtained from the reference specimens, those without discontinuities.

In table 2, theoretical and experimental results of the modulus of elasticity, ultimate tensile strength and relative

Table 3

Set of specimens	Modulus of elasticity (MPa)		Tensile strength (MPa)		Relative uniformity coefficient
	Theoretical	Experimental	Theoretical	Experimental	
CK20	33583	31545	500	488	0.831
CK22	31983	31338	500	503	0.862
CK24	30530	30424	500	489	0.873
CK40	33583	33186	375	397	0.643
CK42	29632	30723	375	380	0.665
CK44	26512	28225	375	369	0.702

uniformity coefficient for reinforced specimens with Kevlar fabric are presented.

In table 3, theoretical and experimental results of the modulus of elasticity, ultimate tensile strength and relative uniformity coefficient for reinforced specimens with Kevlar and carbon fabric are presented.

In tables 1-3, the relative uniformity coefficient was calculated with the help of experimental data obtained at each specimen set.

Conclusions

The properties of composites are influenced by the unevenness that results from the technological processes used for obtaining and processing of these materials. Material defects, irregular distribution of reinforcement, variation in the proportion of its volume, have the effect of lowering the loading capacity. The uniformity coefficient is an indicator that considers the influence of various factors on the mechanical behaviour of materials. The main parameters that influence the uniformity coefficient are: the volume ratio of reinforcement in the area of least strength, the volume ration of the reinforcement in the rest of the material, the size of the area of least strength and the ratio between the modulus of elasticity of the fibres and elastic modulus of the matrix.

Analysis of experimental data highlights the following conclusions:

- the tensile strength decreases with the number of interrupted layers, thus, the tensile strength for the specimens with two interrupted layers is at 80% of the tensile strength from the reference specimens, while the specimens with four interrupted layers have a tensile strength of 60% in comparison with the reference specimens;
- the modulus of elasticity decreases with the increase of interrupted layers area. Theoretically, specimens with no length of interrupted layers have the same modulus of elasticity with the reference specimens (without discontinuities). The Young modulus decreases for specimens with a length of 2 cm of discontinued layers and further decreases for the specimens with a length of discontinued layers of 4 cm. It should be noted that this decrease is greater for specimens that have 4 interrupted layers than for specimens which have 2 interrupted layers;
- in the case of specimens with discontinuities, the relative uniformity coefficient has subunit values. It is noted that with increasing the length of the interruption zone of the layers, a growth of the relative uniformity coefficient occurs. This can be explained by the decrease in the modulus of elasticity with increasing the length of the interrupted area of the layers. Lower values of the relative uniformity coefficient are observed for specimens with 4 interrupted layers than for specimens which have 2 interrupted layers. This is explained by the fact that specimens with 4 interrupted layers in comparison with specimens which have 2 interrupted layers, the decrease in tensile strength is higher, in terms of proportion than the decrease in the modulus of elasticity.

Values of the uniformity factor close to 1 show that the composite material is homogeneous, seamless in reinforcement distribution, the material being close to the reference material, while lower values indicate the presence of defects. Although it does not show the nature and position of defects, a low value of the uniformity coefficient indicates either the presence of zones where material properties are flawed or that these defects are concentrated in a small area. Under the influence of external forces, a punctual breaking of a reinforcement filler wire determines that the load will be taken over by other fibres. Therefore, the study of the influence of fibre breaking over the uniformity coefficient can be placed where discontinuities length is zero (the case of specimens C20, C40, K20, K40, CK20, CK40). In this case, it is noted that the proportion of reinforcement in the area of least strength, the case of intact fibres, has an important influence on the coefficient of uniformity. In contrast, the proportion of reinforcement in the rest of the volume of material and the ratio between the modulus of elasticity of the fibres and modulus of matrix, for usual values, have minor influences on the coefficient of uniformity.

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